XXII. *On an Element of Strength in Beams subjected to Transverse Strain, named by the author "The Resistance of Flexure."*—Second Paper.

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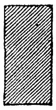
IN my former paper on this subject, I pointed out the existence of an element of strength in beams when subjected to transverse strain, which had been omitted in the generally admitted theory.

The forms of beam employed in the experiments described in that paper were only of two kinds, namely, solid rectangular bars, and open beams or girders.

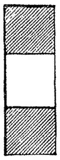
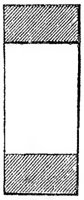
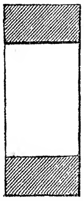
In the experiments given in the present paper I have employed other forms, namely, square bars broken on their sides, square bars broken on their angles, round bars, beams of the **I** section broken with the flanges horizontal, and similar beams broken with the flanges vertical **H**; the object of these experiments being, to elucidate the general bearing of the subject more clearly, and to determine with greater precision than was attempted in my former paper, the laws which govern this resistance.

The following are the results of the experiments in which, for the purpose of more easy reference, I have numbered the several forms of section in continuation of those described in my former paper; and have also included the results of those experiments.

Summary of Experiments on Transverse Strength. Solid and open beams. Length of bearing 60 inches.

Number and form of section.	Total depth of beam.	Depth of metal.	Distance between the bars.	Breadth of bar.	Total sectional area.	Breaking weight.
	in.	in.	in.	in.	sq. in.	lbs.
No. 1 .... 	2·015	2·015	Nil.	·975	1·965	1664
	2·020	2·020	Nil.	·980	1·980	1888
	2·073	2·073	Nil.	1·030	2·135	2084
	2·040	2·040	Nil.	·990	2·020	1916
Mean......	2·012	2·012	Nil.	·994	2·025	1888
No. 2 .... 	2·54	1·98	·56		2·01	2188
	2·53	1·98	·55		2·00	2748
	2·49	1·98	·51		1·96	2412
	2·50	1·95	·55		1·95	2524
Mean......	2·51	1·97	·54	1·005	1·98	2468

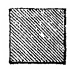
Summary of Experiments on Transverse Strength (continued).

Number and form of section.	Total depth of beam.	Depth of metal.	Distance between the bars.	Breadth of bar.	Total sectional area.	Breaking weight.
	in.	in.	in.	in.	sq. in.	lbs.
No. 3 .... 	3·02	2·04	·98		2·02	3028
	3·00	2·00	1·00		2·00	3224
	3·00	1·99	1·01		1·98	3112
	3·00	1·99	1·01		1·98	2972
Mean......	3·01	2·01	1·00	·995	2·00	3084
No. 4 .... 	3·99	1·99	2·00		2·00	4204
	4·00	1·97	2·03		1·96	4260
	3·99	1·94	2·05		1·96	4204
	4·01	1·97	2·04		1·99	4745
Mean......	4·00	1·97	2·03	1·005	1·98	4353
No. 5 .... 	4·02	2·98	1·04		2·287	5050
	4·05	3·01	1·04		2·290	5125
	4·05	3·01	1·04		2·290	4985
	4·04	3·04	1·00		2·420	5405
Mean......	4·04	3·01	1·03	·771	2·322	5·141
No. 6......... 	4·02	1·50	2·52		2·26	5212
	4·05	1·50	2·55		2·27	5125
	4·03	1·47	2·56		2·19	4845
	4·06	1·45	2·61		2·20	5405
Mean......	4·04	1·48	2·56	1·507	2·23	5147
No. 7 .... 	4·05	1·55	2·50		2·38	5685
	4·10	1·59	2·51		2·45	6525
	4·08	1·57	2·51		2·38	5965
	4·05	1·53	2·52		2·32	5825
Mean......	4·07	1·56	2·51	1·525	2·38	6000

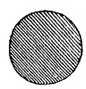
Summary of Experiments on Transverse Strength. Square and round bars of one inch sectional area.

Length of bearing 60 inches.

Square bars broken on their sides.

Number and form of section.	Depth.	Breadth.	Sectional area.	Breaking weight.
	in.	in.	sq. in.	lbs.
No. 8 …… 	1·010	1·020	1·030	505
	1·010	1·025	1·035	505
	1·010	1·020	1·030	561
	1·020	1·025	1·045	533
	1·000	1·020	1·020	533
Mean ………	1·010	1·020	1·032	527

Cylindrical bars.

Number and form of section.	Mean diameter.		Sectional area.	Breaking weight.
	in.		sq. in.	lbs.
No. 9 …… 	1·145		1·030	519
	1·113		·972	505
	1·115		·976	449
	1·118		·981	449
	1·120		·985	449
Mean ………	1·122		·989	474

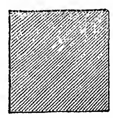
Square bars broken on their angles.

Number and form of section.	Depth.	Side of square.	Sectional area.	Breaking weight.
	in.		sq. in.	lbs.
No. 10 …… 	1·442	1·020	1·040	449
	1·467	1·037	1·076	421
	1·450	1·025	1·050	449
	1·428	1·010	1·020	449
	1·428	1·010	1·020	477
Mean ………	1·443	1·020	1·041	449

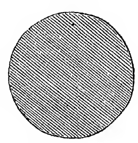
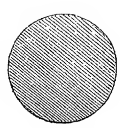
Summary of Experiments on Transverse Strength. Square and round bars of about four inches sectional area.

Length of bearing 60 inches.

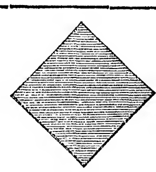
Square bars broken on their sides.

Number and form of section.	Depth.	Breadth.	Sectional area.	Breaking weight.
	in.	in.	sq. in.	lbs.
No. 11. 	1·985	2·020	4·010	3303
	1·990	2·015	4·010	3303
	2·010	2·010	4·040	3443
	2·000	1·990	3·980	3863
Mean	1·996	2·009	4·010	3478

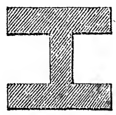
Cylindrical bars.

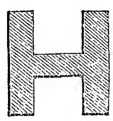
Number and form of section.	Mean diameter.	Breadth.	Sectional area.	Breaking weight.
			sq. in.	lbs.
No. 12. 	2·52		4·987	4283
	2·52		4·987	4283
	2·52		4·987	4003
	2·51		4·948	4003
Mean	2·52		4·977	4143
No. 13. 	2·20		3·801	3068
	2·20		3·801	2988
	2·19		3·767	3388
	2·20		3·801	3228
	2·19		3·767	2988
Mean	2·20		3·787	3132

Square bars broken on their angles.

Number and form of section.	Depth.	Side of square.	Sectional area.	Breaking weight.
	in.		sq. in.	lbs.
No. 14. 	2·835	2·005	4·020	3128
	2·842	2·010	4·040	3268
	2·842	2·010	4·040	2848
	2·820	1·994	3·976	2708
Mean	2·835	2·005	4·020	2988

Summary of Experiments on Transverse Strength. Compound Sections.
Length of bearing 48 inches.

Number and form of section.	Total depth.	Depth of metal in flanges.	Distance between flanges.	Breadth of flanges.	Breadth of middle rib.	Total breadth.	Sectional area.	Breaking weight.
	in.	in.	in.	in.	in.	in.	sq. in.	lbs.
No. 15. 	1·97	·99	·98	1·44	·55	1·99	2·51	3310
	2·00	1·00	1·00	1·50	·47	1·97	2·47	3560
	2·01	1·01	1·00	1·54	·48	2·02	2·52	3735
	2·08	1·11	·97	1·54	·53	2·07	2·81	3910
	2·07	1·06	1·01	1·50	·52	2·02	2·67	4528
	2·07	1·02	1·05	1·57	·47	2·04	2·57	4563
	2·06	1·04	1·02	1·56	·53	2·09	2·71	4423
Mean	2·04	1·03	1·00	1·53	·50	2·03	2·60	4004

Number and form of section.	Total depth.	Depth of centre rib.	Breadth of flanges.	Breadth of centre rib.	Total breadth.	Sectional area.	Breaking weight.
	in.	in.	in.	in.	in.	sq. in.	lbs.
No. 16. 	1·97	·50	·98	1·00	1·98	2·43	2368
	1·96	·48	1·00	·96	1·96	2·42	2288
	2·05	·55	1·10	·92	2·02	2·76	3128
	2·04	·51	1·02	1·00	2·02	2·59	2568
	2·06	·50	1·06	·98	2·04	2·67	2420
	2·05	·50	1·02	1·02	2·04	2·60	2648
	2·05	·52	1·04	1·00	2·04	2·63	2568
Mean	2·02	·51	1·03	·98	2·02	2·59	2569

The neutral axis having been shown in my former paper to be in the centre of gravity of the section, we are enabled to test the accuracy of the existing theory, by comparing the resistance at the outer fibres or particles of each of the forms of beam, calculated upon that theory, with the actual tensile strength of the metal obtained by direct experiment.

In any bar or beam, supported at the ends and loaded in the centre,—

Let f represent the ultimate tension*,
l the length,
W the weight applied in the centre,
d the depth,
and x any variable distance from the neutral axis.

Then $\frac{fx}{d}$ will be the tension at the distance x, and according to the principle of Leibnitz, the sum of all these resistances at the moment of rupture will be

$$\int \frac{fx^2}{d}\, dx\,;$$

* In those materials in which the resistance to compression is less than that of tension, f must be taken to represent the ultimate resistance to compression.

and including the equal resistance to compression

$$2\int\frac{fx^2}{d}\,dx,$$

which taken between the limits $x=0$ and $x=d$, becomes

$$\frac{2fd^2}{3}=\tfrac{1}{4}l\text{W}.$$

In the case of rectangular bars, if the breadth $=b$, this expression becomes

$$\tfrac{2}{3}fbd^2=\tfrac{1}{4}l\text{W}. \quad \ldots \ldots \quad (1.)$$

In girders or bars of other forms, if $y=$ the double ordinate corresponding with the distance x, the general expression will be

$$\int\frac{fx}{d}yxdx\,;$$

and when the form of section is symmetrical above and below the neutral axis,

$$2\int\frac{fx}{d}yxdx=\tfrac{1}{4}l\text{W}.$$

From this general expression we obtain the following for the several forms experimented upon:—

In the square bar broken on its angle, if

$d=$ half the depth,

$y=2(d-x)$,

the complete integral of this between the limits $x=0$ and $x=d$ will be

$$\frac{fd^3}{3}=\tfrac{1}{4}l\text{W}. \quad \ldots \ldots \quad (2.)$$

In like manner, in round bars, if

$d=$ half-depth or radius,
$y=2\sqrt{(d^2-x^2)}$,
$\pi=3{\cdot}1416\ldots$,

then the complete integral is

$$\frac{\pi fd^3}{4}\tfrac{1}{4}l\text{W}. \quad \ldots \ldots \quad (3.)$$

In the open beam, the expression for the resistance is the same as in the rectangular bar, except that here denoting the half-depth by D, and the half-distance between the bars by d, the expression

$$2bf\int\frac{x}{d}xdx$$

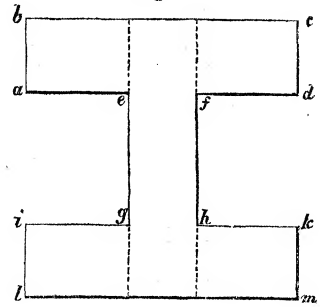
must be taken between the limits $x=d$ and $x=\text{D}$, which gives

$$\frac{2bf}{3}\left(\frac{\text{D}^3-d^3}{\text{D}}\right)=\tfrac{1}{4}l\text{W}. \quad \ldots \ldots \quad (4.)$$

In the case of the section No. 15, broken with the flanges horizontal (see fig. 1),

$D=$ depth.
$b=$ breadth of the centre rib.
$b'=$ breadth of the flanges $ae+fd$.
$d=$ half-distance of the flanges.

Fig. 1.



The expression for the centre rib is

$$\tfrac{2}{3}fbD^2,$$

and for the flanges

$$\frac{2b'f}{3}\left(\frac{D^3-d^3}{D}\right);$$

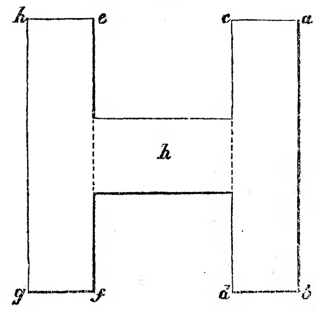
and consequently the resistance to the whole section will be

$$\frac{2f}{3}\left(bD^2+\frac{b'(D^3-d^3)}{D}\right)=\tfrac{1}{4}lW. \qquad (5.)$$

In like manner, for section No. 16, broken with the flanges vertical (see fig. 2),

$d=$ half-depth of the flange $abcd$.
$b=$ width of the two flanges $=he+ca$.
$d'=$ depth of the centre rib.
$b'=$ breadth of the centre rib between the flanges.

Fig. 2.



Then $\dfrac{2fbd^2}{3}=$ resistance of the flanges,

and $\dfrac{2fb'd'^3}{3d}=$ resistance of centre rib;

and consequently the total resistance will be

$$\frac{2f}{3}\left(bd^2+\frac{b'd'^3}{d}\right)=\tfrac{1}{4}lW. \qquad (6.)$$

With these formulæ we are enabled to calculate the resistance of the outer fibre under this generally accepted theory, in each of the sections.

The following Table shows the results:—

	Form of section.	Length of bearing. in.	Breaking weight. lbs.	Value of f, or the calculated resistance at the outer fibre.
No. 6.	Open girder	60	5147	25,271
No. 7.	Open girder	60	6000	27,908
No. 4.	Open girder	60	4339	28,032
No. 3.	Open girder	60	3119	31,977
No. 2.	Open girder	60	2468	35,386
No. 5.	Open girder	60	5141	37,408
No. 1.	Solid rectangular 2×1 inches	60	1888	41,709
No. 8.	Square 1×1 inch	60	527	45,630
No. 9.	Round bar 1 inch area.	60	474	51,396
No. 10.	Square bar broken diagonally	60	449	53,966

Compound Sections.

	Form of section.	Length of bearing. in.	Breaking weight. lbs.	Value of f, or the calculated resistance at the outer fibre.
No. 15.	I Section, flanges horizontal .	48	4008	37,508
No. 16.	⊢⊣ Section, flanges vertical . .	48	2569	43,358

Solid bars of 4 inches sectional area and upwards.

No. 11.	Square bar broken on its side .	60	3478	39,094
No. 12.	Round bar $2\frac{1}{2}$ inches diameter .	60	4143	39,560
No. 13.	Round bar $2\frac{1}{4}$ inches diameter .	60	3132	44,957
No. 14.	Square bar broken on its angle .	60	2988	47,746

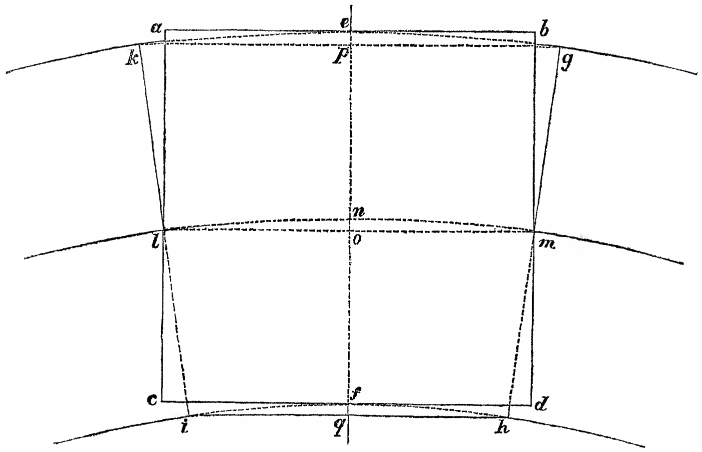
It will be seen from these results, that the apparent resistance at the outer fibre, computed on the principles of this theory, varies from 25,271 lbs. to 53,966 lbs.; while the tensile strength of the metal, as obtained by experiments on direct tension, averages only 18,750 lbs. This discrepancy and variation will be found to arise from the omission of the resistance consequent on the molecular disturbance accompanying curvature.

In my former paper a formula was given by which the difference between the tensile strength and the apparent resistance at the outer fibre could be computed, approximately, in solid rectangular beams and open girders. I now propose to trace the operation of the resistance of flexure, considered as a separate element of strength, and to show its effect in each of the above forms of section.

The theory at present acted upon, proceeds on the assumption that there are only two resistances in a beam, namely, tension and compression; but this supposition fails to account, not only for the strength, but also for the visible changes of figure which arise under transverse strain.

If *abdc* (fig. 3) represent the centre portion of a solid rectangular beam before any

Fig. 3.



strain is applied, *kghi* is the figure which this portion will assume when subjected to

transverse strain, the beam being supposed to be supported at the centre *f*, and loaded at its extremities.

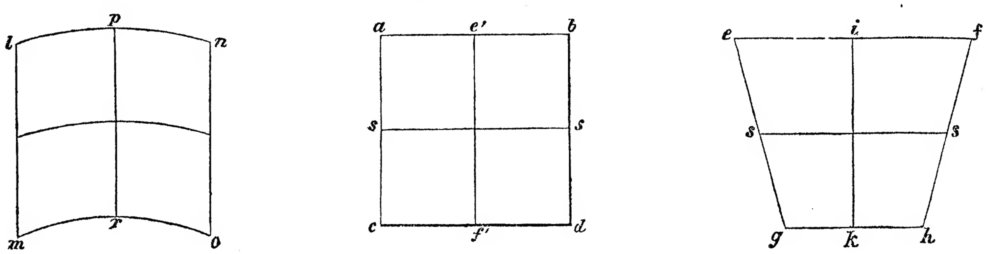
In this change of figure it will be observed that there are three effects:—

First, an extension of the fibres or particles, commencing at the neutral axis *lnm*, and increasing to the upper portion of the beam.

Secondly, a compression of the fibres or particles from the neutral axis to the lower portions of the beam; and

Thirdly, the planes or surfaces *alc* and *bmd* are forced downwards to the distance *ep*, *no* and *fq*.

There are, in fact, two distinct changes of figure:—



There is the change produced by the tension and compression, which, if acting alone, would result in the figure *efhg*; and there is the change produced by curvature, which, if acting alone, would result in the figure *lpnorm*. The effect produced by the curvature is, to cause the sides or planes *bd* and *ac* to descend parallel to themselves; the effect produced by the tension and compression is, to cause these planes to turn about the neutral axis. The combination of these effects is necessary to produce the figure which a beam assumes when placed under transverse strain; and the changes of figure point out distinctly the nature of the resistances. For as it was shown by the measurements taken in the experiments on the neutral axis, that the lines or planes corresponding to *ac* and *bd* remained straight, whatever was the amount of their angular motion, it follows that the tensions and compressions will increase in an arithmetical ratio from the neutral axis to the outer portions of the beam. But the effect of flexure causes the planes corresponding to *ac* and *bd* to descend an equal extent throughout their surfaces; the resistance to this change of figure will therefore be a force distributed evenly over the whole surface.

If *abcd* were a series of horizontal laminæ, these two changes of figure might be obtained separately; *efhg* being the result of the strains applied in the direction of the length, and *lnom* that of a strain applied at right angles to the length.

But if the laminæ are all united together, the elastic reaction of the mass causes certain fixed relations to be established between the curvature and the angles formed by the planes which were at right angles to the length, prior to the strain being applied.

Of these relations, it is sufficient for the present purpose to point out that which subsists between the degree of extension and compression, and the amount of curvature.

Referring again to fig. 3, if b represents any point in the upper surface of a solid beam, before strain is applied, and g the same point when loaded, br* will vary directly as rg. But rg represents the difference between the extension of the fibre, at or nearest the neutral axis, and that at the outer portion of the beam; therefore the resistance to flexure will vary directly as this difference.

In the case of the open beam, the resistance to flexure being only due to that of the bar deflected, whereas the ultimate deflection of the beam is equal to that of a solid beam of the same total depth, the resistance of flexure in the open beam will be to that of the solid beam, at the moment of rupture, as the depth of the bar to the half-depth of the beam; and this is also proportional to the difference between the extension of the fibres nearest the neutral axis, and those at the outer portion of the beam.

The foregoing consideration of the subject, therefore, points out the following properties as belonging to the resistance of flexure:—

1st. That it is a resistance acting in addition to the direct extension and compression.

2nd. That it is evenly distributed over the surface, and consequently (within the limits of its operation) its points of action will be at the centres of gravity of the half-section.

3rd. That this uniform resistance is due to the lateral cohesion of the adjacent surfaces of the fibres or particles, and to the elastic reaction which thus ensues between the portions of a beam unequally strained.

4th. That it is proportional to, and varies with, the inequality of strain between the fibres or particles nearest the neutral axis and those most remote.

We are enabled, under the above-mentioned conditions, to arrive at the relation between the straining and resisting forces in any of the forms of section experimented upon, as resulting from the combined effect of the resistances of tension, compression and flexure.

Using the same letters as before to represent the tension, weight, length, depth, &c., let $\varphi =$ the resistance of flexure acting as a force evenly spread over the surface of the section.

Then, instead of the expression $\frac{fx}{d}$, as representing the resistance at the distance x, we shall have, according to the preceding view, the expression

$$\frac{fx}{d}+\varphi,$$

and these forces acting as before, the moment will be

$$\left(\frac{fx}{d}+\varphi\right)x.$$

The sum of these moments, including those above and below the neutral axis, will be

$$2\int\left(\frac{fx}{d}+\varphi\right)xdx,$$

which, taken between the limits $x=0$ and $x=d$, becomes

$$2(\tfrac{1}{3}f+\tfrac{1}{2}\varphi)d^2=\tfrac{1}{4}l\mathrm{W}.$$

* r, which is not represented in the figure, is the intersection of the lines bm, pg.

Taking $y=$ the double ordinate corresponding to the distance x, the general expression, when the sections are symmetrical above and below the neutral axis, will be

$$2\int\left(\frac{fx}{d}+\varphi\right)yxdx=\tfrac{1}{4}l\text{W}.$$

From this general expression the following are obtained for the several forms experimented upon:—

First, in the case of the square or rectangular bar,

$$2(\tfrac{1}{3}f+\tfrac{1}{2}\varphi)bd^2=\tfrac{1}{4}l\text{W}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

For the square when broken angleways,

$$(\tfrac{1}{3}f+\tfrac{2}{3}\varphi)d^3=\tfrac{1}{4}l\text{W}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

For the round bars,

$$\left(\frac{\pi f}{4}+\frac{4\varphi}{3}\right)d^3=\tfrac{1}{4}l\text{W}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.)$$

For the open bar, since the resistance to flexure depends on the inequality of extension between the part nearest and that most remote from the neutral axis, if $d'=$ the depth of the bar, and D the half-depth of the beam, the resistance to flexure at the moment of rupture will be $\varphi\frac{d'}{\text{D}}$, or multiplied by $d'b$,

$$=\frac{d'^2}{\text{D}}b\varphi;$$

and this resistance acting at the distance $\text{D}-\frac{d'}{2}$, we have, for the whole resistance,

$$2b\left\{\frac{(\text{D}^3-d^3)f}{3\text{D}}+\frac{d'^2}{\text{D}}\left(\text{D}-\frac{d'}{2}\right)\varphi\right\}=\tfrac{1}{4}l\text{W}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10.)$$

In the case of section No. 15 (fig. 1), broken with the flanges horizontal, the expression for the centre part will be

$$2(\tfrac{1}{3}f+\tfrac{1}{2}\varphi)b\text{D}^2;$$

and for the flanges,

$$2b'\left\{\frac{(\text{D}^3-d^3)f}{3\text{D}}+\frac{d'^2}{\text{D}}\left(\text{D}-\frac{d'}{2}\right)\varphi\right\};$$

and consequently for the whole section,

$$2(\tfrac{1}{3}f+\tfrac{1}{2}\varphi)b\text{D}^2+2b\left\{\frac{(\text{D}^3-d^3)f}{3\text{D}}+\frac{d'^2}{\text{D}}\left(\text{D}-\frac{d'}{2}\right)\varphi\right\}=\tfrac{1}{4}l\text{W}. \quad . \quad . \quad . \quad (11.)$$

And lastly, for section No. 16 (fig. 2), broken with the flanges vertical, the expression for the flanges will be

$$2(\tfrac{1}{3}f+\tfrac{1}{2}\varphi)bd^2;$$

and for the centre part,

$$2(\tfrac{1}{3}f+\tfrac{1}{2}\varphi)\frac{b'd'^3}{d};$$

and therefore, for the whole section,

$$2(\tfrac{1}{3}f+\tfrac{1}{2}\varphi)\left(bd^2+\frac{b'd'^3}{d}\right)=\tfrac{1}{4}l\text{W}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12.)$$

These formulæ, applied to the several forms of beams experimented upon, give the following equations:—

No. 1.	$\cdot 67062 f + 1\cdot 0059\ \varphi = 28320$
No. 2.	$1\cdot 0425\ f + 1\cdot 1813\ \varphi = 37020$
No. 3.	$1\cdot 4473\ f + 1\cdot 3388\ \varphi = 46260$
No. 4.	$2\cdot 3297\ f + 1\cdot 4698\ \varphi = 65295$
No. 5.	$2\cdot 0625\ f + 2\cdot 2043\ \varphi = 77115$
No. 6.	$3\cdot 0564\ f + 1\cdot 3512\ \varphi = 77205$
No. 7.	$3\cdot 2227\ f + 1\cdot 5059\ \varphi = 90000$
No. 8.	$\cdot 1734\ f + \cdot 2601\ \varphi = 7905$
No. 9.	$\cdot 13867 f + \cdot 23541 \varphi = 7110$
No. 10.	$\cdot 12519 f + \cdot 25039 \varphi = 6735$
No. 11.	$1\cdot 3336\ f + 2\cdot 0009\ \varphi = 52170$
No. 12.	$1\cdot 5708\ f + 2\cdot 6666\ \varphi = 62145$
No. 13.	$1\cdot 0454\ f + 1\cdot 7746\ \varphi = 46980$
No. 14.	$\cdot 9484\ f + 1\cdot 8968\ \varphi = 44820$
No. 15.	$1\cdot 281\ f + 1\cdot 126\ \varphi = 48048$
No. 16.	$\cdot 711\ f + 1\cdot 066\ \varphi = 30828$

If the metal were of precisely uniform strength, f and φ would be precisely constant quantities, and their value might be obtained from any two of these equations; but as considerable variation occurs in the strength, even in castings of the same dimensions, and as a reduction of strength, per unit of section, is known to arise when the thickness of the metal is increased, the values of f and φ will necessarily vary, and can only be ascertained in each experiment by first establishing the ratio they bear to each other.

For this purpose the first ten experiments may be used, all of which were made of metal of from three-quarters to one inch in thickness, the mean tensile strength of which was ascertained by direct experiment to be 18750 lb. per inch.

Using this value of f in each case, we have

No. 1. $$\varphi = \frac{28320 - \cdot 67062 \times 18750}{1\cdot 0059} = 15654$$

No. 2. $$\varphi = \frac{37020 - 1\cdot 0425 \times 18750}{1\cdot 1813} = 14748$$

No. 3. $$\varphi = \frac{46260 - 1\cdot 4473 \times 18750}{1\cdot 3388} = 14284$$

No. 4. $$\varphi = \frac{65295 - 2\cdot 3297 \times 18750}{1\cdot 4698} = 14667$$

No. 5. $$\varphi = \frac{77115 - 2\cdot 0625 \times 18750}{2\cdot 2043} = 17442$$

No. 6. $\varphi=\frac{77205-3\cdot0564\times18750}{1\cdot3512}=14725$

No. 7. $\varphi=\frac{90000-3\cdot2227\times18750}{1\cdot5059}=19640$

No. 8. $\varphi=\frac{7905-1\cdot734\times18750}{\cdot2601}=17892$

No. 9. $\varphi=\frac{7110-\cdot13867\times18750}{\cdot23541}=19158$

No. 10. $\varphi=\frac{6735-\cdot12519\times18750}{\cdot25039}=17523.$

Mean value of $\varphi=16573$.

Ratio of f to φ, as 1 to ·847.

If we use the following experiments of Mr. Hodgkinson's on the breaking weight of inch bars, of which the tensile strength was ascertained by direct experiment, the following results are obtained:—

Description of iron.	Transverse strength of the bar, 54 inches bearing.	Tensile strength per inch of the metal.	Computed value of ϕ*.	Ratio of f to ϕ.
	lbs.	lbs.	lbs.	
Carron iron No. 2, cold blast	476	16,683	14,582	1 to ·874
Carron iron No. 2, hot blast	463	13,505	15,999	1 to 1·185
Carron iron No. 3, cold blast	446	14,200	14,617	1 to 1·029
Carron iron No. 3, hot blast	527	17,755	14,621	1 to ·824
Devon iron No. 3, hot blast	537	21,907	14,393	1 to ·657
Buffery iron No. 1, cold blast	463	17,466	13,358	1 to ·765
Buffery iron No. 1, hot blast	436	13,434	14,588	1 to 1·086
Coed-Talon iron No. 2, cold blast	413	18,855	9,732	1 to ·516
Coed-Talon iron No. 2, hot blast	416	16,676	11,347	1 to ·682
Low Moor iron No. 3, cold blast	467	14,535	15,528	1 to 1·066
Mean	464	16,502	14,076	1 to ·853

These results indicate that the ratio between the resistance of tension and the resistance of flexure varies in different qualities of metal, and this supposition appears confirmed by other experiments on rectangular bars, given in the 'Report of the Commissioners on the Application of Iron to Railway Structures.' The mean result, however, accords nearly with that of my own experiments, and shows that the resistance of flexure, computed as a force evenly distributed over the section, is almost nine-tenths of the tensile resistance.

Employing this ratio of the values of f and φ, and applying it to the equations result-

* The sign ϕ was employed in my former paper to indicate the difference between the tensile force and the apparent resistance at the outer fibre. It is here used as the measure of the resistance considered as acting evenly over the surface; hence the value of ϕ, as here employed, will be two-thirds of the difference between the tensile resistance and the apparent resistance at the outer fibre in the rectangular bar.

ing from the experiments on the tensile strength of the metal, as derived from each form of section, the deduced values of f will be as follows:—

	Form of Girder.	
No. 1.	Solid rectangle	f=17,971
No. 2.	Open girder	f=17,582
No. 3.	Open girder	f=17,442
No. 4.	Open girder	f=17,882
No. 5.	Open girder	f=19,058
No. 6.	Open girder	f=18,070
No. 7.	Open girder	f=19,659
No. 8.	Square bar, 1 inch, section broken on its side	f=19,399
No. 9.	Round bar, 1 inch, section	f=20,236
No. 10.	Square bar, 1 inch, section broken on its angle	f=19,213
No. 11.	Square bar, 4 inches, section broken on its side	f=16,644
No. 12.	Round bar, $2\frac{1}{2}$ inches diameter	f=15,902
No. 13.	Round bar, $2\frac{1}{4}$ inches diameter	f=17,778
No. 14.	Square bar, 4 inches, sectional area broken on its angle	f=16,878
No. 15.	Compound section, flanges horizontal	f=20,942
No. 16.	Compound section, flanges vertical	f=18,460

The results thus obtained, though not perfectly regular, are within the limits of the variation exhibited by the metal, as shown by the experiments on direct tension given in the former paper.

If the results be classified,—

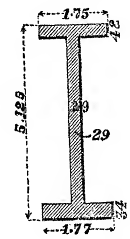
The mean tensile strength, as obtained from the open girders, Experiments Nos. 2, 3, 4, 5, 6 and 7, is	18282
From the solid bar, No. 1	17971
From the inch bars, square, round, and square bars broken diagonally, Nos. 8, 9 and 10	19616
From the bars of 4 inches sectional area, square, round, and square bars, broken diagonally, Nos. 11, 12, 13, and 14	16800
From the compound sections, in which the metal was half an inch thick	19701

The variation in strength, as exhibited between the small and the large bars, is in accordance with the experiments made by Lieut.-Colonel JAMES, and recorded in the 'Report of the Commissioners upon the Application of Iron to Railway Structures.'

The results obtained in all these varieties of form of section being so far satisfactory, it appeared desirable to test the application of the formulæ to other known experiments, of which the following may be given as examples:—

First, an experiment made by Mr. HODGKINSON, and given in TREDGOLD'S 'Treatise on Cast Iron,' 4th edit. The form of the beam is as shown in the figure. In this case,—

$D = 2{\cdot}5625.$
$b = {\cdot}29.$
$b' = 1{\cdot}47$, mean.
$d = 2{\cdot}1573.$
$d' = 1{\cdot}405$, mean.
$l = 54.$
$W = 6678.$



And employing the formula used for the section No. 16, we have

$$2{\cdot}8631f + 2{\cdot}3476\varphi = 90153;$$

and if φ be taken at nine-tenths of f,

$$f = 15086.$$

The tensile strength thus computed, accords very closely with the quality of metal employed by Mr. HODGKINSON in that and other experiments made by him at that time on various forms of girders.

In the Reports on the 'Strength and other properties of Metals for Cannon,' made by the Officers of the Ordnance Department of the United States Government, some experiments are given upon the transverse strength of square and round bars of cast iron. These experiments were made with very great care by Major WADE, for the purpose of testing various qualities of metals and modes of treatment, by frequent re-casting, and by keeping the metal for various periods of time under fusion. From each experiment, a constant is derived for the purpose of comparing the relative strengths of metal; and in endeavouring to obtain the constant for round iron, Major WADE has employed the usually accepted theory of the transverse strain. He appears, however, to have found that the formula is defective, for he observes at page 21 of the Report,—"A trial was made with cylindrical bars in lieu of square bars. These generally broke at a point distant from that pressed, and the results were so anomalous that the use of them was soon abandoned. The formula by which the strength of the round bars is computed, appears to be not quite correct; for the unit of strength in the round bars is uniformly much higher than in the square bars cast from the same kind of iron."

The following are the experiments on the round bars, with those on the square bars from the same metal; and it will be seen, that if the tensile strength of the metal be computed by the formula here given, including the resistance to flexure, the discrepancy pointed out by Major WADE disappears; and the tensile resistance, whether obtained for the round or the square bars, agrees very nearly with that derived from the experiments on direct tension under like circumstances.

Experiments on the Transverse Strength of Cast Iron, made by the Officers of the Ordnance Department of the United States Government.

Square bars. Length of bearing 20 inches.

Description of iron.	Number of experiment.	Hours in fusion.	Breadth.	Depth.	Breaking weight.	Tensile resistance calculated from the formula, including resistance of flexure.
			in.	in.	lbs.	lbs.
Franklin iron. Second fusion...	9	$1\frac{1}{3}$	2·025	2·058	12,712	18,920
	10	2	2·000	2·054	12,712	19,233
	11	$2\frac{2}{3}$	1·994	2·008	13,950	22,149
	12	$2\frac{2}{3}$	1·989	2·013	11,700	18,531
Third fusion ...	79	$2\frac{3}{4}$	1·975	1·999	14,569	23,566
	80	$2\frac{3}{4}$	1·977	2·008	13,387	21,440
Third fusion ...	21	0	2·025	1·980	12,987	20,882
	22	0	2·020	1·990	13,365	21,330
	23	1	2·030	1·990	15,363	24,396
	24	1	2·030	1·990	14,616	23,211
	25	2	2·020	2·050	13,788	20,735
	26	2	2·050	2·070	14,850	21,582
	27	3	2·025	2·060	16,056	23,852
	28	3	2·035	2·020	16,722	25,708
Third fusion ...	29	$\frac{1}{2}$	1·978	2·003	12,994	20,904
	30	$1\frac{1}{2}$	1·930	2·003	15,300	25,226
	31	3	1·977	2·028	15,862	24,904
	32	$3\frac{3}{4}$	2·010	2·008	16,172	25,473

Round bars. Length of bearing 20 inches.

Description of iron.	Number of experiment.	Hours in fusion.	Diameter.	Breaking weight.	Tensile resistance computed from the formula, including resistance of flexure.
			in.	lbs.	lbs.
Franklin iron... Second fusion...	37	1	1·975	7,920	20,711
	38	2	1·950	9,270	25,188
	39	3	1·953	9,481	25,644
	40	4	1·975	7,920	20,711
Third fusion ...	81	$\frac{1}{2}$	2·415	16,425	23,493
	82	$1\frac{1}{2}$	2·420	18,141	25,788
	83	$2\frac{1}{2}$	2·420	20,419	29,093
	84	$2\frac{3}{4}$	2·420	19,997	28,425
	85	$2\frac{3}{4}$	2·420	18,225	25,907
Third fusion ...	33	$\frac{1}{2}$	1·960	10,437	27,927
	34	$1\frac{1}{2}$	1·970	8,665	22,833
	35	3	2·000	11,112	27,984
	36	$3\frac{3}{4}$	1·960	10,606	28,378

The tensile strength of the same metal, as ascertained by direct experiment, is thus stated at page 44 of the Report:—

No. of fusion.	Franklin iron.		
	6-pounder gun, 3rd fusion.	Gun No. 61, 2nd and 3rd fusions.	Mean.
	lbs.	lbs.	lbs.
1st	25,969	15,861	20,915
2nd	29,143	20,420	24,781
3rd	27,755	24,383	26,569
4th	30,039	25,773	27,906
7th	......	29,690	

Although not bearing directly on the subject of this paper, I cannot refrain from calling attention to the extraordinary development of strength in cast iron, obtained in the experiments made by the United States Government. It will be seen, on referring to the Reports from which the above Tables are taken, that by frequent recasting and keeping the metal under fusion during periods of from three to four hours, an increase of 60 per cent. is obtained; and that the strength of the American iron so treated is more than double that of English under the usual mode of manufacture.

The general accordance presented between the value of the tensile resistance, obtained by direct experiment, and that computed by means of the foregoing formulæ in so many varieties of form of section, is such as to confirm the view here taken of the laws which govern the action of the resistance of flexure.

It remains only to refer to two points connected with it, first, as to the ratio it bears to the tensile resistance. If the metal were homogeneous and the elasticity perfect, it is probable that the resistance of flexure would be precisely equal to the tensile resistance, instead of bearing the ratio of nine-tenths, as found by experiment. It is evident, however, that it varies in different qualities of metal, and that the tensile resistance does not bear a constant ratio to the transverse strength.

The following Table, taken from Major WADE's valuable Reports, shows, that with the same metal and different modes of casting, an increase of transverse strength is obtained, while a decrease takes place in the tensile resistance.

Guns.	Transverse strength.		Tensile strength.		Specific gravity.	
	Bar cut from gun.	Bar cast separate.	Bar cut from gun.	Bar cast separate.	Bar cut from gun.	Bar cast separate.
6-pounder gun, No. 6 ...	8415	9,880	30,234	29,143	7·196	7·263
6-pounder gun, No. 8 ...	9233	9,977	31,087	30,039	7·278	7·248
8-inch gun, No. 64	8575	10,176	26,367	24,583	7·276	7·331
Mean	8741	10,011	29,229	27,922	7·250	7·281
Proportional	1·000	1·145	1·000	·955	1·000	1·004

From the above, it appears that with a decrease of about one-twentieth in the tensile strength, there is an increase of nearly three-twentieths in the transverse strength.

It is easy to conceive also, that though the resistance of flexure might be supposed to maintain nearly the same proportion to the tensile resistance in bodies similarly consti-

tuted, as for example crystalline substances, yet great variation may be expected to occur between crystalline and malleable and fibrous substances.

The only other point to be referred to is, as to the limit of action of the resistance of flexure. It appears evident that in all the simple solid sections, the points of action of the resistance of flexure are the centres of gravity of the half-section; while in the compound sections it is necessary to compute the centre rib and flanges as for two separate beams in which the resistance of flexure is different, and has its point of action at the centre of gravity of the separate portions.

It would appear that the elastic reaction developes this resistance to the full extent, when the section is such that a straight line may be drawn from every point at the outer portion to every point at the neutral axis within the section; but that if the form of section is such that straight lines drawn from the outer fibres, or particles, to the neutral axis fall without the section (as in the case in the compound sections, Nos. 15 and 16), then it must be treated as two separate beams, each having that amount of resistance of flexure due to the depth of the metal contained in it.

Resistance of Flexure in Wrought Iron.

Although from the fact, that in a cast-iron beam (the section being a solid rectangle) the neutral axis was found to be at the centre of gravity of the section, it might have been inferred that the same would be found in wrought iron; yet it was considered desirable to ascertain it by actual measurement. For this purpose two beams were taken, one of rolled iron, 7 feet 6 inches long, 6 inches in depth, and $1\frac{1}{4}$ inch in breadth; the other of hammered iron, 8 feet long, $7\frac{1}{4}$ inches in depth, and $1\frac{3}{4}$ inch in breadth. Holes were drilled at about 6 inches on each side of the centre, or point of application of the strain, for the insertion of the pins of the measuring instrument. The holes were six in number, and placed at equal distances from the upper to the lower side of the beam; and the experiments were conducted in the same manner as those made with the cast-iron beam, and described in my former paper.

Experiment for Determination of Neutral Axis.

Wrought Iron Beam (rolled iron).

Depth . . . . 5·93 inches.
Breadth . . . . 1·28 inch.
Length of bearing . . 60 inches.

Beam without weight.	Difference.	Weight applied at centre, 7840 lbs.	Difference.	Weight applied at centre, 11,200 lbs.	Difference.	Weight taken off.	Permanent set.
Micrometer readings.		Micrometer readings.		Micrometer readings.		Micrometer readings.	
1741	+25	1766	+13	1779	−38	1741	
1769	+14	1783	+ 7	1790	−22	1768	−1
1653	+ 5	1658	+ 5	1663	−10	1658	
1787	− 5	1782	+ 2	1784	+ 2	1786	−1
1706	−12	1694	− 8	1686	+22	1708	+2
1746	−23	1723	−11	1712	+38	1750	+4

The same beam with the bearing distance increased to 84 inches.

Beam without weight.	Differ-ence.	Weight applied at centre, 8000 lbs.	Differ-ence.	Weight taken off.	Per-manent set.	Beam without weight.	Differ-ence.	Weight applied at centre, 8000 lbs.	Differ-ence.	Weight taken off.	Per-manent set.
Microm. readings.		Microm. readings.		Microm. readings.		Microm. readings.		Microm. readings.		Microm. readings.	
1757	+36	1793	−36	1757	...	1753	−36	1717	+35	1752	−1
1787	+17	1804	−18	1786	−1	1786	−23	1763	+21	1784	−2
1682	+ 4	1686	− 3	1683	+1	1683	− 9	1674	+ 7	1681	−2
1812	−11	1801	+10	1811	−1	1813	+ 6	1819	− 5	1814	+1
1743	−26	1717	+24	1741	−2	1739	+24	1763	−21	1742	+3
1779	−39	1740	+40	1780	+1	1779	+35	1814	−38	1776	−3

Experiment for Determination of Neutral Axis.

Wrought Iron Beam (hammered iron).

Depth . . . . 7·25 inches.
Breadth . . . . 1·75 inch.
Length of bearing . . . . 88 inches.

Beam without weight.	Difference.	Weight applied at centre of beam, 10,266 lbs.	Difference.	Weight applied at centre of beam, 19,226 lbs.	Difference.	Weight applied at centre of beam, 23,706 lbs.	Difference.	Weight taken off.	Permanent set.
Micrometer readings.		Micrometer readings.		Micrometer readings.		Micrometer readings.		Micrometer readings.	
1560	+24	1584	+21	1605	+18	*1623	−61	1562	+ 2
1599	+18	1617	+10	1627	+ 8	1635	−34	1601	+ 2
1624	+ 7	1631	+ 4	1635	− 2	1633	−15	1618	− 6
1643	− 5	1638	− 6	1632	− 8	1624	+ 6	1630	−13
1456	−12	1444	−14	1430	−20	1410	+27	1437	−19
1401	−22	1379	−25	1354	−29	1325	+52	1377	−24

Although the extensions and compressions are only about half that of cast iron, and consequently the liability to error in the measurements is increased in proportion, yet the experiments point out that the position of the neutral axis in wrought iron, like that of cast, is at the centre of gravity of the section, and that the action is the same in both materials, excepting as to the amount of the extensions and compressions with a given strain.

The formula $2(\frac{1}{2}f+\phi\frac{1}{2})bd^2=\frac{lW}{4}$, given in the former part of the paper for cast iron, will therefore apply to wrought iron also.

The relative values of f and ϕ are not so readily ascertained in wrought iron, because the material yields by bending and not by fracture. And another point requires consideration, namely, that the ultimate compressive strain which wrought iron is capable

* Previous to these measurements being taken, a weight of 14,093 lbs. was applied on the end, equal to 28,186 lbs. on the centre of the beam, but was reduced to 23,706 lbs. in the centre. The elasticity of the beam had, however, been overcome, as shown by the permanent set and by subsequent experiments on the same beam.

of sustaining is little more than half its ultimate tensile strength. But although there exists this disproportion as regards the ultimate resistances by tension and compression, the force required to overcome the elasticity of the material is nearly the same, whether applied as a compressive or a tensile strain; the difference being, that the force which overcomes the elasticity when applied as a compressive strain, leads to the destruction or distortion of the material; while in the case of the tensile strain, the elasticity may be overcome long before the material yields by absolute rupture.

The following experiments, made in Woolwich Dockyard by Professor BARLOW, show the relative weights which overcome the elasticity of the metal when applied transversely, as compared with the weight necessary to produce the same result when applied by direct tension.

Three bars, numbered 5, 6 and 7, each 2 inches square, were tested by direct tension; and the strain which was just sufficient to overcome the elasticity of the iron, was found to be—

	Tons per square inch of section.
No. 5. Re-manufactured iron	9·5
No. 6. Re-manufactured iron, from old furnace-bars .	8·25
No. 7. New bar by Messrs. GORDON	10·00

Bars of the same quality of metal having been subjected to transverse strain in a bearing of 33 inches, the weights which overcame their elasticity, when so applied, were as follows:—

	Tons.	Mean.
No. 5	3·00	3·00
No. 5	3·00	
No. 6	2·50	2·25
No. 6	2·00	
No. 7	3·00	2·83
No. 7	2·50	
No. 7	3·00	

Using the tensile resistance in each case for f, and obtaining the value of φ from the formula

$$\varphi=\frac{lW}{4bd^2}-\tfrac{2}{3}f,$$

	Tons.	Tons.
No. 5 . . .	$\varphi=6{\cdot}04$	$f=9{\cdot}50$
No. 6 . . .	$\varphi=3{\cdot}78$	$f=8{\cdot}25$
No. 7 . . .	$\varphi=5{\cdot}01$	$f=10{\cdot}00$
Mean . .	$\varphi=4{\cdot}94$	$f=9{\cdot}25$

Mean ratio . . $f:\varphi::1:{\cdot}53$.

In addition to these, six other experiments are given on iron, of the quality of No. 7, which are as follows:—

Number of experiment.	Length of bearing.	Breadth.	Depth.	Weight which overcame the elasticity.	Value of ϕ computed from the formula.
	in.	in.	in.	tons.	tons.
8	33	1·9	2·0	2·5	3·65
9	33	1·9	2·0	2·5	3·65
10	33	1·9	2·0	2·5	3·65
11	33	1·5	3·0	4·5	4·33
12	33	1·5	3·0	4·5	4·33
13	33	1·5	2·5	3·25	4·77
				Mean	4·06

Mean ratio of f to ϕ : 1 to ·406.

The general mean of these results appears therefore to show, that the resistance of flexure in wrought iron, considered as a force acting evenly over the surface, is nearly equal to one half of the tensile resistance. It is, however, desirable that further experiments should be made with this material.

Appendix to the foregoing pages. *By* PETER BARLOW, *Esq., F.R.S.*

Received March 25,—Read April 2, 1857.

Application of the preceding principles to Beams and Girders of non-symmetrical section.

IN beams of symmetrical section the neutral axis corresponds with the centre of gravity, because in that case all the direct forces above and below that point are necessarily equal. But when the section is non-symmetrical, it is requisite, in order to determine the position of the neutral axis, to find that point in the section in which this condition has place, viz. that point below which the sum of all the direct resistances to tension and curvature are equal to all those above that point due to compression and curvature; then to find the sum of the moments of these resistances separately; and finally, to equate them with the straining force.

The double-flanged girder with unequal flanges forms a good subject for testing the general application of the principles developed in the preceding pages. In such a girder, let

a denote the whole depth of the girder;
m the thickness of the middle web;
d the depth of the bottom flange;
d' the depth of the upper flange;
b the breadth of the former, *minus* m;
b' the breadth of the latter, *minus* m;
x the required distance of the neutral axis from the bottom of the girder;
x' the distance of the same from the upper face of the girder;
t the tensile resistance of the lower fibres;
c the corresponding resistance to compression of the upper fibres.

Now if we consider the centre rib as carried through the two flanges, the sum of the direct resistances due to the tension of the metal in the middle rib below x will be $\frac{1}{2}mxt$,

and the sum of those due to curvature or change of figure, $mx\phi$; or calling $\phi=t$*, the whole direct resistance of this central web below x will be $\frac{3}{2}mxtb$. Again, since the direct tensile resistance of the unsupported flange varies as the distance from the neutral axis, if we consider x as representing a constant quantity, and y any variable distance from the neutral axis, $bt\int\frac{ydy}{x}$ (taken between $y=x$ and $y=x-d$) becomes $\left(d+\frac{d^2}{2x}\right)bt$, the sum of all the direct tensile resistances; the resistance to change of figure being expressed simply by dbt.

The total direct resistance below the neutral axis is therefore

$$\left(\tfrac{3}{2}mx+2bd-\frac{d^2}{2x}b\right)t.$$

In like manner, the total direct resistance to compression above the neutral axis is

$$\left(\tfrac{3}{2}mx'+2b'd'-\frac{d'^2}{2x'}b'\right)c,$$

which must be made equal to the former expression.

But we must here observe, that the compression of the upper fibre c, is to the corresponding tension of the lower fibre t, as x' to x; substituting accordingly, rejecting the common factor t, and observing that $x'=a-x$, we find

$$x=\frac{3ma^2+4d'b'a+d^2b-d'^2b'}{6ma+4(db+d'b')}.$$

Having thus determined the position of the neutral axis, we have now to take the moments of these several direct forces both above and below that line, the formula for which are however already given in the preceding pages; that for the lower part of the central web being $\frac{5}{6}m\mathrm{D}^2t$ (D now representing x, the distance above found), and that for the unsupported flange being the same as in the open beam, viz.

$$\frac{\mathrm{D}^3-\overline{\mathrm{D}-d}^3}{3\mathrm{D}}bt+\frac{d}{\mathrm{D}}(\mathrm{D}-\tfrac{1}{2}d)dbt.$$

This latter is, however, reducible to a more convenient form for numerical calculation, viz. to

$$\left(\mathrm{D}-\frac{d^2}{6\mathrm{D}}\right)dbt.$$

We have therefore

$$\tfrac{5}{6}m\mathrm{D}^2t+\left(\mathrm{D}-\frac{d^2}{6\mathrm{D}}\right)dbt=\mathrm{R},$$

the resistance below the neutral axis, and

$$\tfrac{5}{6}m\mathrm{D}'^2c+\left(\mathrm{D}'-\frac{d'^2}{6\mathrm{D}'}\right)d'b'c=\mathrm{R}',$$

the resistance above the neutral axis.

* In the preceding paper, Mr. W. H. BARLOW, by obtaining from experiments a mean value of t, has, by means of his original equation for rectangular bars, *i. e.* $(\frac{2}{3}t+\frac{1}{2}\phi)\mathrm{D}^2=\frac{1}{4}lw$, and his other equations for beams of other forms when broken transversely, endeavoured to find a mean value of ϕ, and he finds the latter to be to the former as about 9 : 10; but from the difficulty of obtaining the mean value of t within certain wide limits, I have not hesitated in assuming t and ϕ equal to each other in the case of cast iron.

But $c:t::\mathrm{D}':\mathrm{D}$. We have, therefore, for the whole resistance above and below the neutral axis,

$$\left(\mathrm{R}+\frac{\mathrm{D}'}{\mathrm{D}}\mathrm{R}'\right)t.$$

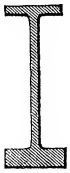
If now we represent by w the breaking weight in any experiment of a beam of given dimensions, and by l the length, or rather the distance between the props, there is obtained the expression

$$\left(\mathrm{R}+\frac{\mathrm{D}'}{\mathrm{D}}\mathrm{R}'\right)t=\tfrac{1}{4}lw,$$

an equation from which, when w and l are given, t may be determined. Or if t be previously experimentally determined, w may be found.

In order to submit these equations to the test of experience, we have selected from the valuable and extensive series of experiments, by EATON HODGKINSON, Esq., published in vol. v. of the Memoirs of the Literary and Philosophical Society of Manchester, Second Series, a few experiments in which the girders differ most from each other in section, dimensions, and bearing distance; and the results we obtain from the foregoing formula are given in the following pages, with the form of section and linear dimensions. It will be seen that the value of t, or the direct tensile strength of cast iron, thus obtained, falls generally between the limits of t=14,000 and t=16,000.

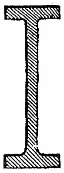
In the Reports of the Commissioners for inquiry into the 'Application of Iron to Railway Structures' (page 9, &c.), there are given the results of about fifty experiments on the direct tensile resistance of one-inch square cast-iron bars, under the direction of Mr. HODGKINSON; these consisted of seventeen different kinds of iron, each set of three bars being of the like quality and manufacture, and in several of these sets, which one might have expected to yield very nearly the same results, the difference is full as great as in the following Table, exhibiting in fact very nearly like numbers. This circumstance will, it is presumed, be considered satisfactory evidence of the general applicability of the principles developed in the preceding pages to cast-iron beams and girders of every variety of section.

(1)* 

Distance of props. 4 feet 6 inches.

- Depth of girder . . . . 5·125 inches.
- Upper flange . . . 1·75 by ·42 inch.
- Lower flange . . . 1·77 by ·39 inch.
- Thickness of centre web . . ·29 inch.
- Breaking weight . . . . . 6678 lbs.

Computed value of t=14578.

(2) 

4 feet 6 inches.

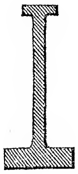
- Depth of girder . . . . 5·125 inches.
- Upper flange . . . 1·74 by ·26 inch.
- Lower flange . . . 1·78 by ·55 inch.
- Thickness of centre web . . ·30 inch.
- Breaking weight . . . . . 7368 lbs.

Computed value of t=14128.

* These numbers are those of the experiments selected from Mr. HODGKINSON's series.

Distance of props.

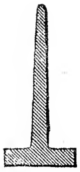
(3)



4 feet 6 inches.

Depth of girder . . . . 5·125 inches.
Upper flange . . . 1·07 by ·30 inch.
Lower flange . . . . 2·1 by ·57 inch.
Thickness of centre web . . ·32 inch.
Breaking weight . . . . . 8270 lbs.
Computed value of t=14005.

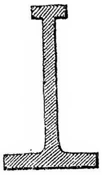
(4)



4 feet 6 inches.

Depth of girder . . . . 5·125 inches.
No upper flange.
Lower flange . . . . 2·27 by ·52 inch.
Mean thickness . . . . . ·405 inch.
Breaking weight . . . . . 8720 lbs.
Computed value of t=13868.

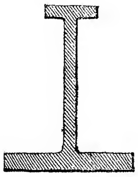
(9)



4 feet 6 inches.

Depth of girder . . . . 5·125 inches.
Upper flange . . . 1·05 by ·34 inch.
Lower flange . . . 3·08 by ·51 inch.
Thickness of centre web . . ·305 inch.
Breaking weight . . . . 10,727 lbs.
Computed value of t=14,765.

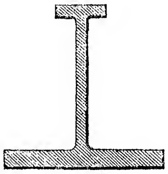
(11)



4 feet 6 inches.

Depth of girder . . . . 5·125 inches.
Upper flange . . . 1·60 by ·315 inch.
Lower flange . . . ·416 by ·53 inch.
Thickness of centre web . . ·38 inch.
Breaking weight . . . . 14,462 lbs.
Computed value of t=14,832.

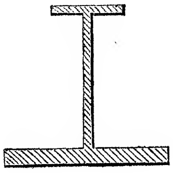
(12)



4 feet 6 inches.

Depth of girder . . . . 5·125 inches.
Upper flange . . . 1·56 by ·315 inch.
Lower flange . . . 5·17 by ·56 inch.
Thickness of centre web . . ·34 inch.
Breaking weight . . . . 16,730 lbs.
Computed value of t=14,181.

(15)

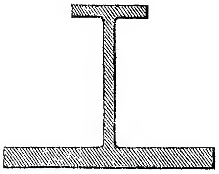


4 feet 6 inches.

Depth of girder . . . . 5·125 inches.
Upper flange . . . 2·35 by ·29 inch.
Lower flange . . 5·43 by ·537 inches.
Thickness of centre web . . ·35 inch.
Breaking weight . . . . 16,905 lbs.
Computed value of t=13,918

Distance of props.

(19)

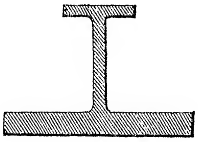


4 feet 6 inches.

Depth of girder	5·125 inches.
Upper flange . . .	2·33 by ·31 inch.
Lower flange . . .	6·67 by ·66 inch.
Thickness of centre web . .	·266 inch.
Breaking weight	26,084 lbs.

Computed value of t=15,474.

(23)

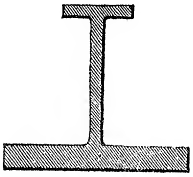


7 feet.

Depth of girder	4·1 inches.
Upper flange . . .	2·25 by ·33 inch.
Lower flange . . .	6·00 by ·74 inch.
Thickness of centre web . .	·40 inch.
Breaking weight	13,543 lbs.

Computed value of t=16,720.

(24)

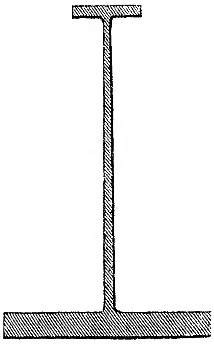


7 feet.

Depth of girder	5·2 inches.
Upper flange . . .	2·25 by ·35 inch.
Lower flange . . .	6·00 by ·77 inch.
Thickness of centre web . .	·34 inch.
Breaking weight	15,129 lbs.

Computed value of t=13,612.

(30)

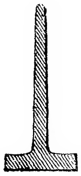


9 feet.

Depth of girder	10·25 inches.
Upper flange	2·1 by ·27 inch.
Lower flange . . .	6·14 by ·77 inch.
Mean thickness of centre web	·27 inch.
Breaking weight	28,672 lbs.

Computed value of t=14,606.

(34)

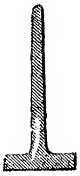


4 feet 6 inches.

Depth of girder	5·125 inches.
No upper flange.	
Lower flange . . .	2·27 by ·46 inch.
Mean thickness of centre web	·37 inch.
Breaking weight	8792 lbs.

Computed value of t=15,374.

(35)



4 feet 6 inches.

Depth of girder	5·125 inches.
No upper flange.	
Lower flange . . .	2·26 by ·47 inch.
Mean thickness of centre web	·352 inch.
Breaking weight	9044 lbs.

Computed value of t=15,980.

Tabulated Results.

Number of HODGKINSON's series.	Upper flange.		Lower flange.		Thickness of centre web.	Depth of girder.	Breaking distance.	Breaking weight.	Computed value of t.
	Breadth.	Depth.	Breadth.	Depth.					
	inches.	inch.	inches.	inch.	inch.	inches.	ft. in.	lbs.	lbs.
1	1·75	·42	1·77	·39	·29	$5\frac{1}{8}$	4 6	6,678	14,578
2	1·74	·26	1·78	·55	·30	$5\frac{1}{8}$	4 6	7,368	14,128
3	1·07	·30	2·10	·57	·32	$5\frac{1}{8}$	4 6	8,270	14,005
4	No upper flange.		2·27	·52	Mean. ·405	$5\frac{1}{8}$	4 6	8,720	13,868
9	1·05	·34	3·08	·51	·305	$5\frac{3}{8}$	4 6	10,727	14,765
11	1·60	·315	4·16	·53	·38	$5\frac{3}{8}$	4 6	14,462	14,832
12	1·56	·315	5·17	·56	·34	$5\frac{3}{8}$	4 6	16,730	14,181
15	2·35	·29	5·43	·537	·35	$5\frac{3}{8}$	4 6	16,905	13,918
19	2·33	·31	6·67	·66	·266	$5\frac{3}{8}$	4 6	26,084	15,474
23	2·25	·33	6·00	·74	·40	4·1	7 0	13,543	16,720
24	2·25	·35	6·00	·77	·34	5·2	7 0	15,129	13,612
30	2·10	·27	6·14	·77	·27	$10\frac{1}{4}$	9 0	28,672	14,606
34	No upper flange.		2·27	·44	Mean. ·37	$5\frac{1}{8}$	4 6	8,792	15,374
35	No upper flange.		2·26	·47	·352	$5\frac{1}{8}$	4 6	9,044	15,980

In the preceding investigation the breaking weight is given from which to determine the tensile resistance; but the usual practical question is, to find the breaking weight, having first ascertained the tensile strength, which is of course simply to reverse the last operation. In the case of small beams, of the kind employed in the foregoing experiments, a sufficiently near approximation, it will be seen, may be obtained by assuming t=14,500 or 15,000, except in peculiar kinds and mixtures of iron; these will generally require a higher number, which must be previously determined.

But it appears from the results of experiments by Mr. HODGKINSON, given at page 111 of the 'Appendix to the Report of the Commissioners on Railway Structures,' and others by Lieut.-Colonel JAMES, R.E., page 251, &c., that a much lower value of t must be taken when the thickness of the casting becomes 2, $2\frac{1}{2}$ or 3 inches, as in large railway girders. Mr. HODGKINSON found, that bars of 1, 2 and 3 inches square, broken on props having the same relative distances, manifested a decrease of strength in the proportion of 1, ·780, ·756; and Colonel JAMES's experiments gave a still greater decrease, viz. of 1, ·794, ·624, and which, by other experiments, he traces to an imperfect crystallization of the interior particles, in consequence probably of the more rapid cooling of the exterior parts. It appears, therefore, that in those large castings commonly employed for railway bridges, it would not be safe to assume t at more than 10,000 lbs.

The large girder broken and reported at page 94 by Mr. HODGKINSON, treated as in the preceding cases, gives t=10,533 lbs. Its length was 45 feet; depth, $29\frac{1}{2}$ inches; the thickness of the lower flange, $2\frac{9}{16}$ inches; and its weight, 18,000 lbs.